

**Projet ANR- Blanc International II -  
SIMI 2 - Science informatique et applications**

**GUWENSHIBIE**

Programme Blanc International II 2012



**Work Package n°2**

Deliverables n°4 and n°5

Image Pre-processing and Restoration

Benchmark and evaluation of restoration tools  
Self controlled algorithms to tune automatically the  
parameters of restoration and enhancement.

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Delivery date: 01 November 2014

## 1. Objectives

Among ancient Chinese manuscripts, some of them show defects due to the aging of paper and ink or due to digitization constraints. Image pre-processing consists of enhancing the images in order to simplify the layout analysis, the extraction of robust features and the strokes retrieval, which is essential to Optical Character Recognition. In the opposite, the image restoration consists of retrieving the exact information lost because of the ink and paper aging. Image restoration is dedicated to historians, palaeographers, codicologist and users who want to study the document in the original layout. For the Guwensibie project, we have two objectives :

- Digitally enhance the image by pre-processing algorithms in order to simplify the image recognition steps.
- Try to segment the image to separate character shapes from the paper without loss of information about the character patterns.

## 2. Study of the type of degradations on Chinese rolls

Chinese manuscripts show specific degradations that we have classified into three categories :

### 2.1 Paper Degradations

The paper of Ancient manuscripts show many defects which appear as noise in the background of the images. It is the main degradation observed in these manuscripts. It leads to ghost patterns in the background of the digital image and colour noise which can be confused with the colour of the characters.

Chinese manuscripts have stains due to the humidity which must be removed or reduced in order to make the character segmentation easier.

Ancient Chinese manuscripts can present some holes or missing parts which cannot be filled by digital inpainting if the size of the hole is too large. Digital inpainting can fill thin holes like scratches. Generally, ancient Chinese manuscripts show large holes which cannot be restored by digital inpainting.

Unlike Latin or Arabic manuscripts, Chinese ancient rolls are handwritten mostly in only one side. Consequently, we rarely notice ink bleed-through degradation when the ink can bleed through to the other side so that the characters from the reverse side appear on the foreground.

### 2.2 Ink Degradations

These degradations can lead to broken or touching characters, which are the major causes of OCR errors. The ink fading and disappearance is the most frequent degradation observed on Chinese manuscripts. This loss of information is caused by the aging of inks. Once the document is digitized, these degradations become a part of the document. The ink fading leads to a complete or partial loss of the information

in some parts of the characters because the ink has the same colour as that of the paper. It decreases the performance of the OCR and makes more difficult the segmentation of characters. Restoring the ink fading consists of enhancing the remaining information about parts of characters which are no more visible and to extrapolate lost information.

The degradation of inks due to aging also impacts the quality of the digital images of the characters. The noise along the contours of the characters and also inside the characters must be reduced in order to facilitate the segmentation process and the feature extraction.

### 2.3 Degradation due to digitization constraints

These degradations appear during the digitization process and depend on the digitizer and the work of the operator. For the Guwenshibie project, the digitization process is nearly optimal. The rolls are perfectly aligned, and the image does not require any deskew operation. The manuscripts are maintained flat so that there are no geometrical distortions like curvatures, or warping. Illumination is uniform along all the rolls. There is no pre-processing operation to restore degradations due to the digitization process.

### 2.4 Summary of degradations

Table 1 summarizes the different types of degradation observed in Chinese manuscripts and figure 1 is a good illustration of these defects. We do not develop pre-processing tools for the degradations which are not observed into Chinese manuscripts of the project.

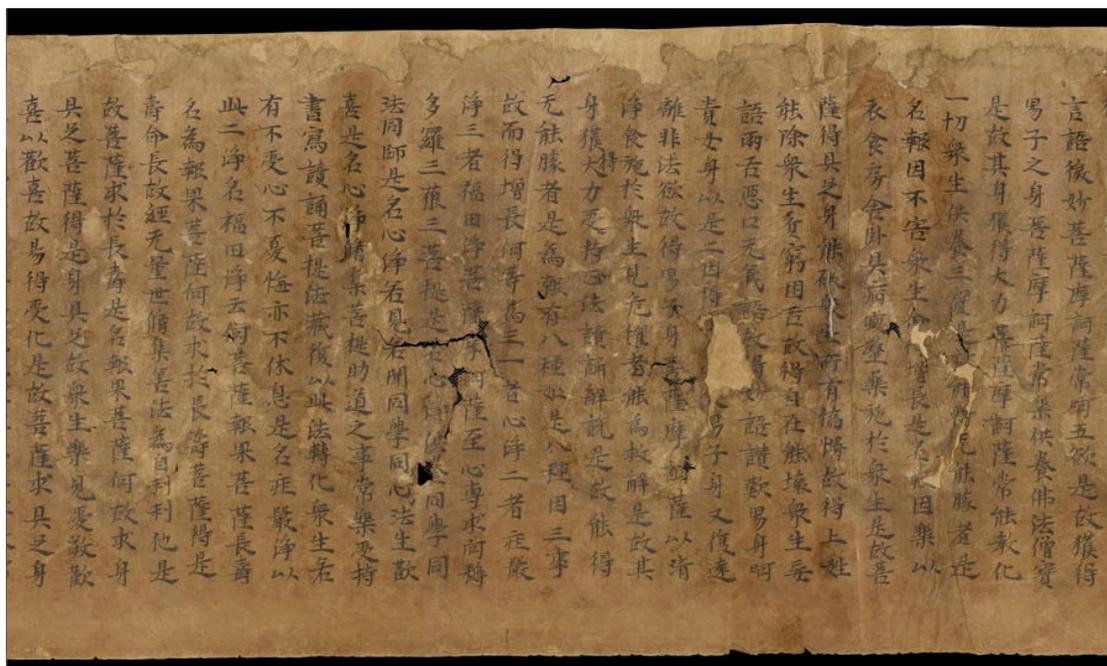


Figure 1: Frequent degradations observed on Chinese historical manuscripts.

<b>Degradation</b>	<b>Solution</b>
<b>Degradations of the papers</b>	
Noise in the background	<i>Denoising by PDE regularization like TV, anisotropic diffusion, Non Local Means, Bilateral filtering...</i>
Paper stains	<i>Colour analysis by morphology, colour clustering, colour enhancement</i>
Missing parts	<i>No attempt to fill holes by digital inpainting techniques or by sparse analysis because we know in advance that these approaches fail when the holes are too large. Too much information is lost for a reconstruction..</i>
ink bleed-through	<i>No such degradation observed</i>
<b>Degradations of the ink</b>	
The ink fading and the ink disappearance	<i>Colour analysis by morphology, colour clustering, colour enhancement, Retinex,</i>
Ink noises	<i>Denoising by PDE regularization like TV, anisotropic diffusion, Non Local Means, Bilateral filtering, Coherence-Enhancing filtering</i>
<b>Degradation due to digitization constraints</b>	
Non illumination and no colour constancy	<i>No such degradation observed</i>
Out of focus blurring which require image sharpening, chocking or deblurring	<i>No such degradation observed</i>
Geometrical degradations (paper warping, skew, rotation, curvature...)	<i>No such degradation observed</i>

Table 1: List of the degradations and the associated algorithms to enhance the image.

In conclusion, the main pre-processing operations to experiment are the image denoising and colour segmentation or enhancement in order to separate the Chinese characters around stains or when the ink is fading. We favour methods which are not based on machine learning and which do not require prior information or a model because such approaches can fail on handwritten manuscripts with complex shapes having too many spatial variations.

### 3. Experimentation on images of Chinese manuscripts

The objective consists to develop an autonomous system for image restoration and enhancement already developed in the LIRIS lab. We have tested all existed on digitized Chinese manuscripts. We must first benchmark and evaluate existing tools developed in the LIRIS and in the literature, compare the results and find the default parameters for each method. We also propose to automatically tune the parameters according to the image contents when it is feasible. We apply the algorithms on different images from IDP website showing severe degradations. As the size of the images are very large, we just illustrate in this report, the effects of each algorithm on a very small part of the original image, at the original scale presented in Figure 2.

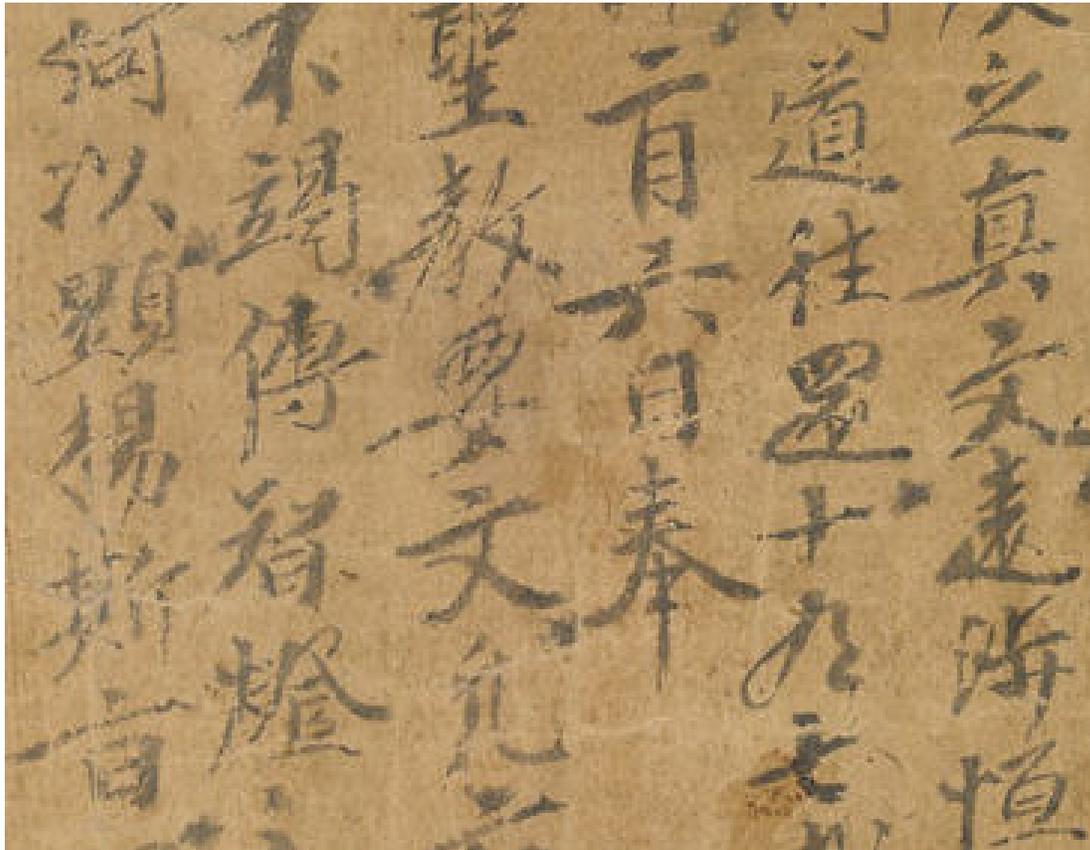


Figure 2: Small part of the original Image zoomed at 100%. This illustration shows the typical degradations due to the noise of the ink and the background and the ink fading.

### 3.1 Denoising of ink and background paper

We do not propose approaches which require prior information or based on machine learning which require training. Model driven restoration techniques are efficient only if the degradations can be observed statistically several times in the same configuration. In this case we can train a Neural Network or build dictionaries with a sparse analysis to learn repeated degradations for each configuration of pixels and its neighbourhood. The model driven approach works perfectly well for printed characters having predictable and repetitive structures which can be learned.

In the case of handwritten manuscripts, the spatial variability is too high and the number of possible local structures is too great that it is difficult to train a system or find statistical repetitions to build dictionaries.

We focus only on data driven approaches which do not require any training or prior knowledge on character patterns.

We detail three different pixel-driven approaches to reduce noise:

- *Weighted Averaging approaches*
- *Approaches based on Total Variation*
- *Approaches based on Heat equation*

Most of these approaches can be solved by variational methods and expressed mathematically by Partial Differential Equations of PDE.

#### 3.1.1 *Weighted Averaging models*

These approaches use a weighted averaging with pixels  $\tilde{x}_i$  found locally in the neighbourhood of the central pixel  $\tilde{x}$  or non locally in similar parts of the image (1). The weights  $w(\tilde{x}_i)$  can use spatial information or/and colour information in the colour space.

$$(1) \quad \tilde{x} = \frac{\sum_i \tilde{x}_i \times w(\tilde{x}_i)}{\sum_i w(\tilde{x}_i)}$$

Different models use the weighted averaging approach to reduce noise and segment the image like the Bilateral filter, Non Local Mean and the MeanShift. According to the weights calculation and the information used for the averaging, the weighted averaging produces completely different results.

### a) Bilateral filter

A bilateral filter is a non-linear edge-preserving and noise-reducing smoothing filter for images. The intensity value at each pixel in an image is replaced by a weighted average of intensity values from nearby pixels having similar colours (2). These weights depend on the Euclidean distance of pixels from the central pixel and on the colour distance in the colour space. A Gaussian distribution is used to adjust the computation of the weights. The standard deviations  $\sigma_R$  and  $\sigma_S$  adjust the weights for the range colour distance in the colour space and the Euclidean distance in the spatial space, respectively [Tomasi98]. It simultaneously sharpens character contours and reduce the noise of the background.

$$(2) \quad \bar{x}_p = \frac{\sum_i \bar{x}_i \times K_{\sigma_R}(\bar{x}_i - \bar{x}_p) \times K_{\sigma_S}(i - p)}{\sum_i K_{\sigma_R}(\bar{x}_i - \bar{x}_p) \times K_{\sigma_S}(i - p)}$$

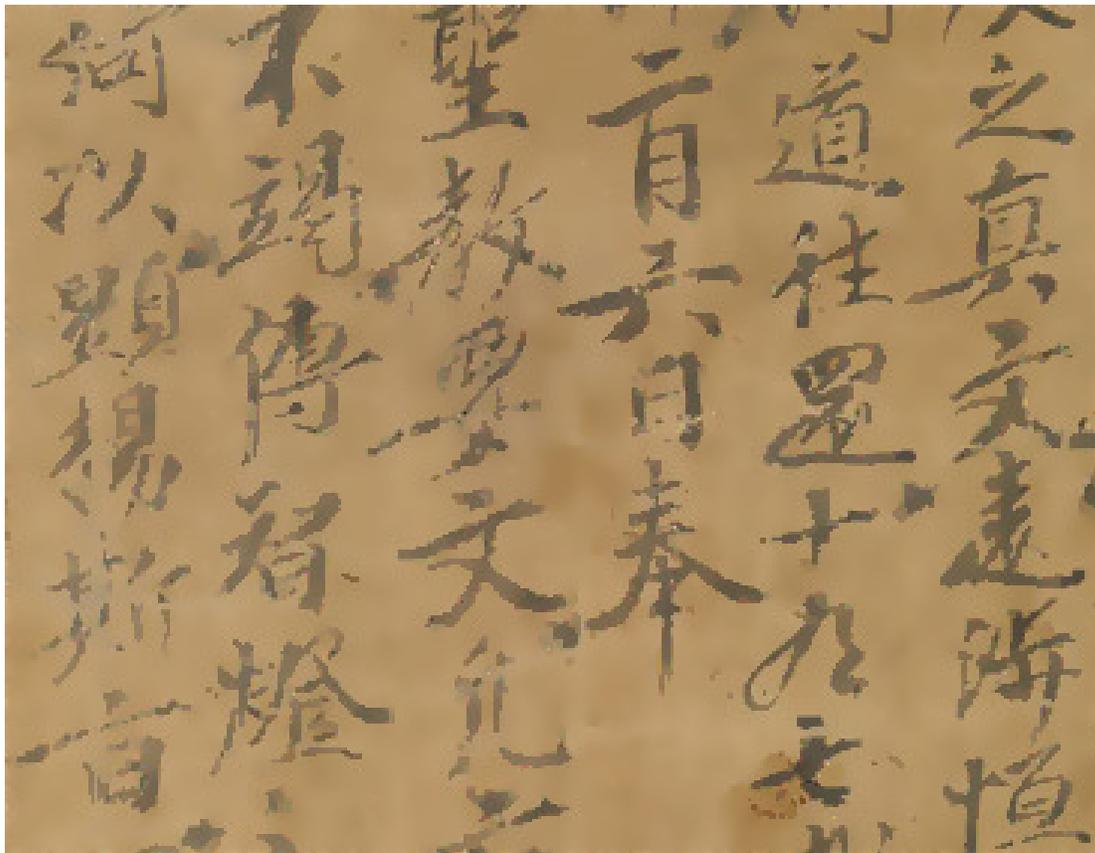


Figure 3: Application of the Bilateral filtering

The bilateral filtering does not smooth the noise along character contours and cannot repair the discontinuities of broken characters. However, it sharpens the character contours. This filter is not advised to restore images of ancient manuscripts because of the loss of information and details necessary for the OCR.

## b) Non Local Means of Buades

[Buades05] introduce the Non Local means which use the image redundancies to reduce noise. The intensity value at each pixel is replaced by a weighted average of intensity values from neighbours having similar neighbourhoods (3). It is similar to the bilateral filtering but with non local information found in neighbourhoods having similar contents. The weights of the averaging depend on the similarity of the contents between the neighbourhood  $N(x_i)$  and the local information  $N(x)$ . The standard deviations  $\sigma_R$  adjust the filtering. The higher  $\sigma_R$  is, the stronger the filtering is.

$$(3) \quad \bar{x} = \frac{\sum_i \bar{x}_i \times K_{\sigma_R}(N(\bar{x}_i) - N(\bar{x}))}{\sum_i K_{\sigma_R}(N(\bar{x}_i) - N(\bar{x}))}$$

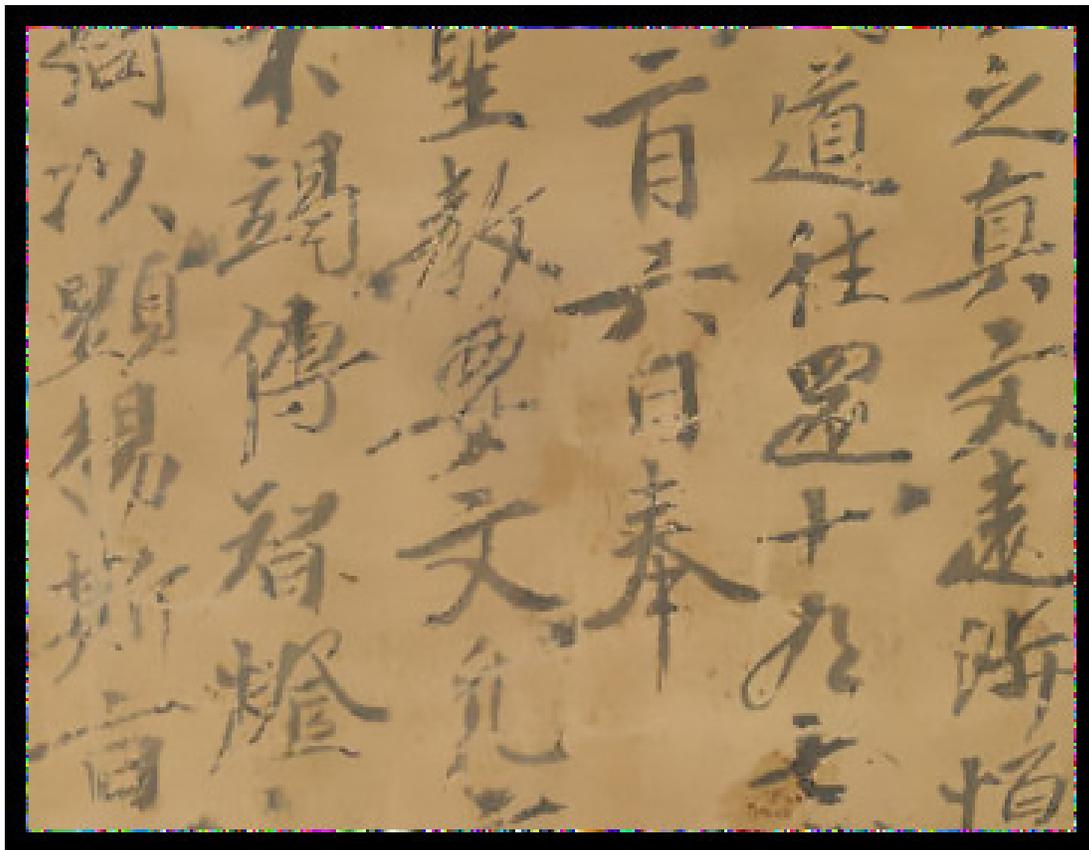


Figure 4: Application of the Non Local Means

Compare to the Bilateral filter, the NLM performs better. However, for ancient manuscripts with complex and non repetitive structures, the NLM fails to find similar neighbourhoods. If the NLM does not find similar neighbourhood locally, we automatically enlarge the search window to a larger area. The best solution will be to search similar neighbourhoods in the whole image. However, in this case, the computation complexity will be too high to process an entire image in reasonable time. Like the Bilateral, the NLM fails to repair discontinuities when the ink is fading. Furthermore, the NLM smooth the contours and preserve all the details necessary for the character recognition.

### 3.1.2 Total Variation

The Total Variation or TV from Rudin, Osher, Fatemi [Rudin92] minimizes the integral of the magnitude of the gradient of the signal and removes unwanted details like noise and speckles while preserving important details such as edges. It corresponds to the application of the following PDE (4). We use the split Bregman optimization to speed up the image processing.

$$\text{Min} \int_{\Omega} \|\nabla I\| dx dy \quad (4) \quad \frac{\partial I}{\partial t} = \text{div} \left( \frac{\nabla I}{\|\nabla I\|} \right)$$

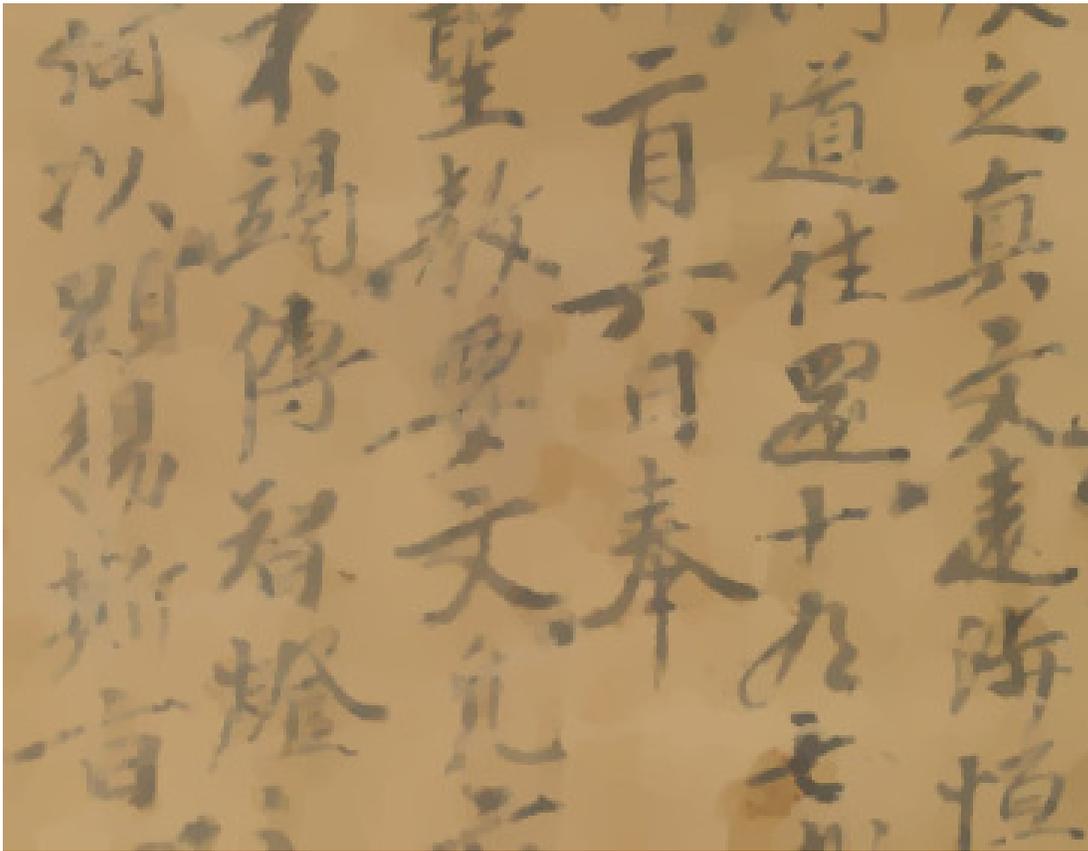


Figure 5 Effect of the TV with split Bregman optimization

Little and thin details of the Chinese characters are not preserved by the TV filtering. Discontinuities of strokes are not restored by the TV. This model also fades the ink and enlarges the stroke discontinuities.

### 3.1.3 Anisotropic Diffusion

Anisotropic diffusion minimizes the integral of the square of the magnitude of the gradient of the signal. The Solution leads to PDE (5) for a scalar diffusivity coefficient and (6) for a matrix diffusivity coefficient.

$$\text{Min} \int_{\Omega} \|\nabla I\|^2 dx dy \quad (5) \quad \frac{\partial I}{\partial t} = \text{div}(c \nabla I) \quad (6) \quad \frac{\partial I}{\partial t} = \text{div}(D \nabla I)$$

#### a) Perona & Malik scalar driven diffusion

The scalar driven diffusion (7), introduced by [Perona90], preserves the edges but does not smooth the contours of the characters. The coefficient of diffusivity is a decreasing curve. It stops the diffusion when the magnitude of the gradient is larger than a predefined threshold K. The parameter K delimits the differences between the noise and the pixels of the contours. We expect that noise have a lower magnitude than the contours of characters.

$$(7) \quad \frac{\partial I}{\partial t} = \text{div}(c(\|\nabla I\|) \times \nabla I) \quad \text{with} \quad C(\|\nabla I\|) = \exp\left(-\frac{\|\nabla I\|^2}{K^2}\right)$$

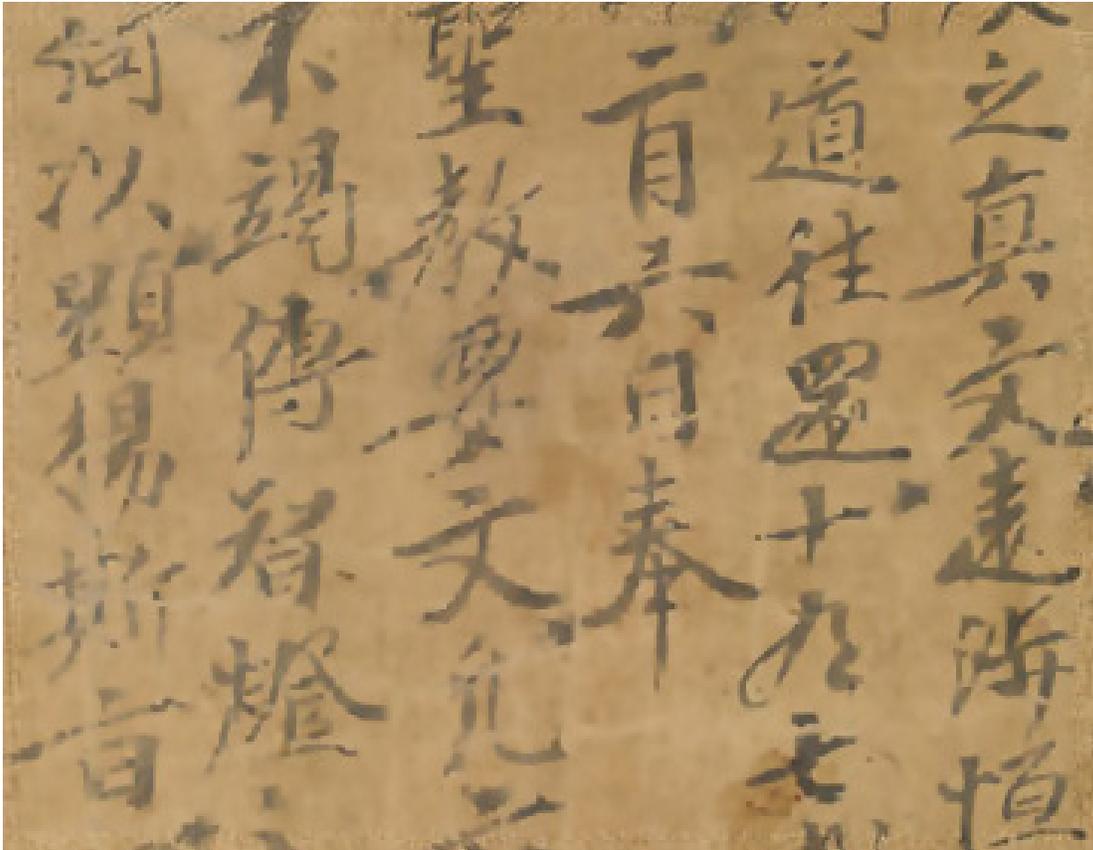


Figure 6: Scalar driven diffusion of Perona&Malik with K=10

Noise and defects remain along the contours and broken characters are not repaired.

## b) Coherence enhancing anisotropic diffusion from Weickert

[Weickert96] introduces a new model which reinforces the coherency of the continuity of the strokes. He was the first to introduce the matrix driven diffusion into the heat equation. The matrix  $D$  of the diffusivity is calculated from the smoothed tensors field. It is oriented to the main direction of the tensors and  $D$  is always anisotropic along this direction (8).

$$(8) \quad \frac{\partial I}{\partial t} = \text{div}(D\nabla I) \quad \begin{cases} D = \mu_+ \times \Theta_- \Theta_-^T + \mu_- \times \Theta_+ \Theta_+^T \\ \Theta_{+/-}, \lambda_{+/-} \text{ eigen vectors and values of Tensors } T_\sigma^p \end{cases}$$

Tensors fields smoothed by Gaussians  $T_\sigma^p = G_\rho \otimes (\nabla(G_\sigma \otimes I) \nabla(G_\sigma \otimes I)^T)$

The parameters of the Weickert model are

$$\begin{cases} \mu_+ = \alpha + (1 - \alpha) \exp\left(\frac{-C}{(\lambda_+ - \lambda_-)^2}\right) \text{ if } \lambda_+ \neq \lambda_- \\ \text{and } \mu_+ = \alpha \text{ else} \\ \mu_- = \alpha \end{cases}$$

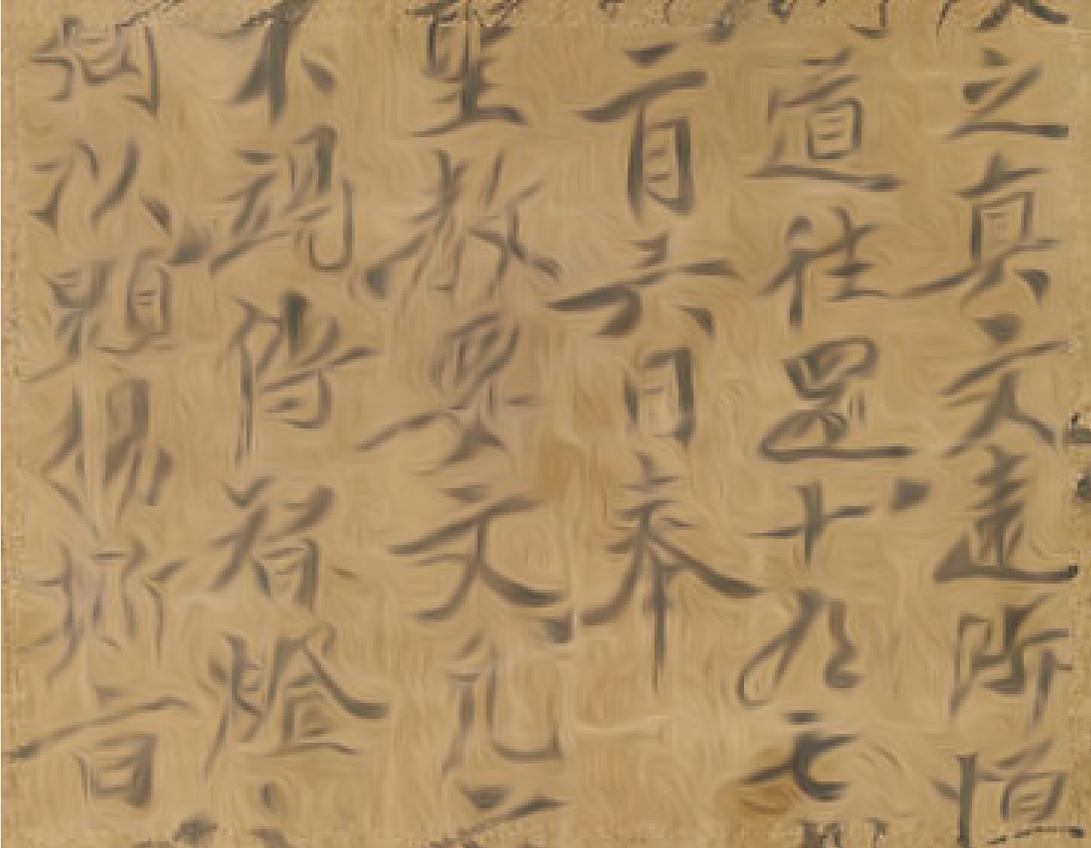


Figure 7: Matrix driven diffusion of Weickert

As expected, the continuities of lines are reinforced, but the endings of the lines may change of direction. Important local information like intersections and micro structures are damaged by this model of restoration.

**c) D. Tschumperle Trace of the Hessian.**

[Tschumperle02] introduces a more efficient model of denoising based on the trace of the Hessian Matrix (9). The efficiency of this model has been proven. It can restore heavily degraded natural images.

$$(9) \quad \frac{\partial I}{\partial t} = \text{trace}(DH) \quad \begin{cases} D = \frac{1}{\sqrt{1+\lambda_+ + \lambda_-}} \times \Theta_- \Theta_-^T + \frac{1}{1+\lambda_+ + \lambda_-} \times \Theta_+ \Theta_+^T \\ \Theta_{+/-}, \lambda_{+/-} \text{ eigen vectors and values of } T_\sigma^p \end{cases}$$

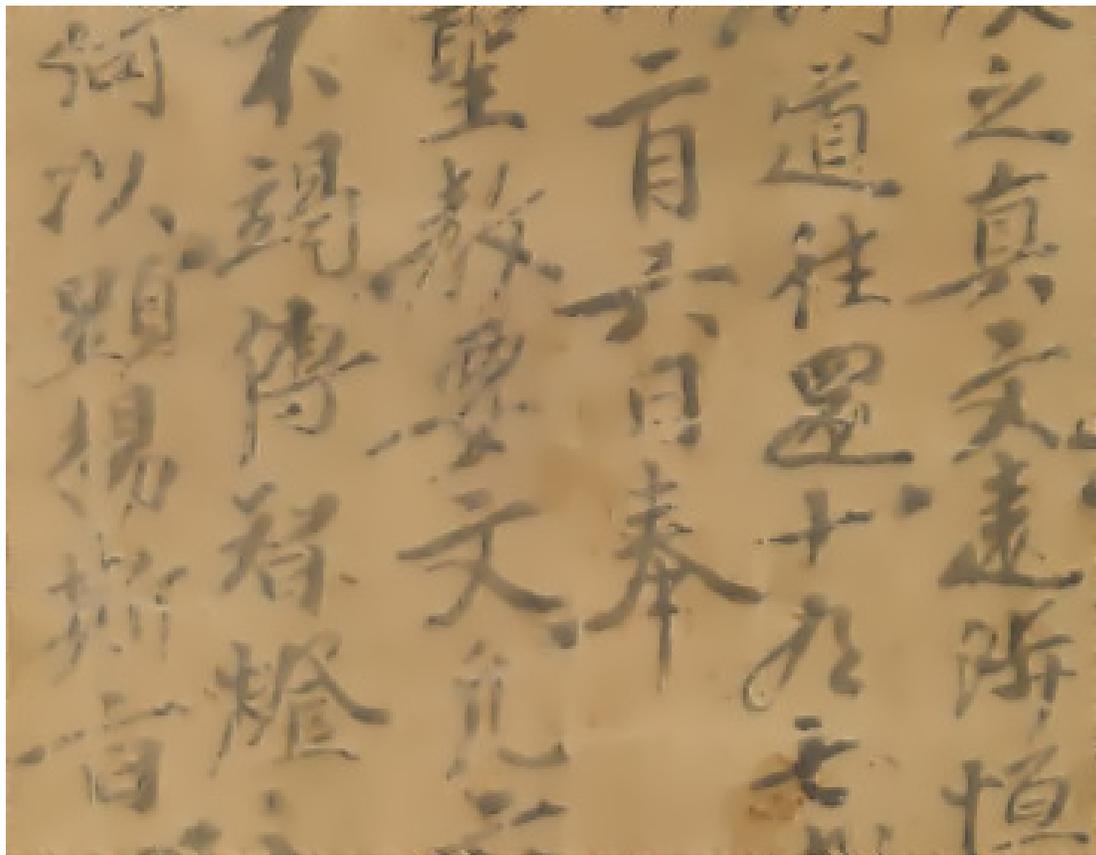


Figure 8: Anisotropic diffusion of D. Tschumperle

Applied on Chinese ancient manuscripts, this model does not repair the discontinuities of strokes, but it reduces efficiently the noise. The main drawback of this model is the loss of information when the ink is fading. However, this model preserves all existing information within the image.

#### d) Anisotropic diffusion which preserves edges and singularities by the LIRIS

Developed exclusively for text denoising, this model seeks to preserve all important information for the OCR, especially singularities like intersections, strokes endings, junctions of strokes. It is a combination of two models, the Perona&Malik model which preserves singularities and the Weickert coherency enhancement model which restores the continuities of lines (10). The resulting model of this combination is a new model which preserves singularities and simultaneously reinforces the continuities of strokes [Dira11]. Two new parameters  $K_+$  and  $K_-$  are necessary to fix.  $K_+$  defines the contours to regularize and  $K_-$  defines the singularities to preserve.

$$10) \frac{\partial I}{\partial t} = \text{div}(D\nabla I) \quad \left\{ \begin{array}{l} D^* = e^{-\left(\frac{\lambda_-}{K_-}\right)} \times \Theta_+ \Theta_+^T + e^{-\left(\frac{\lambda_+}{K_+}\right)} \times \Theta_- \Theta_-^T \\ \Theta_{+/-}, \lambda_{+/-} \quad \text{eigen vectors and values of } T_\sigma^p \end{array} \right.$$

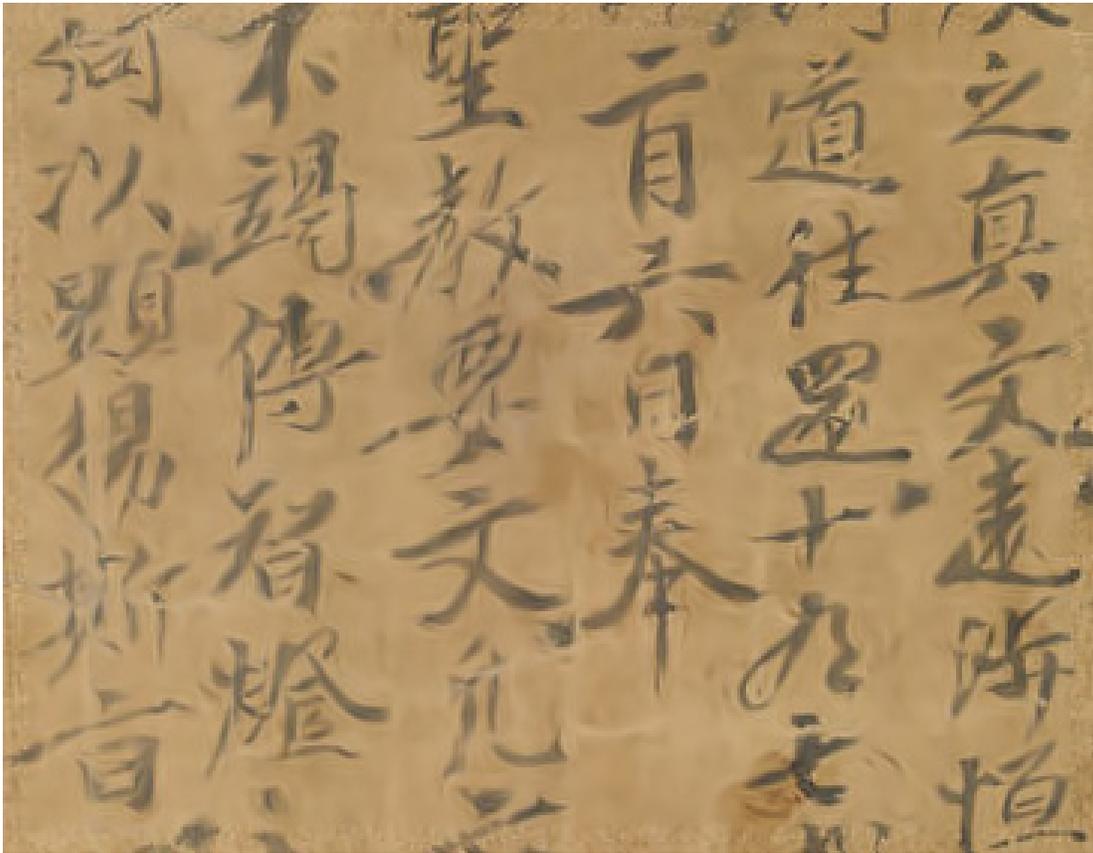


Figure 9: Anisotropic diffusion which preserves singularities and reinforces the continuities of strokes with  $K_+ = 20$  and  $K_- = 100$

This filter preserves the existing information required by the OCR and reconstructs the continuities of strokes. The main drawback is the parameters  $K_+$  and  $K_-$  to fix manually or automatically. The automatic calculation of these parameters has been done for the entire image. The parameters could fail if the background colour change locally around stains for example. Local adaptation of these parameters still does not work properly.

### e) The Beltrami flow

With the edge and singularities preserving diffusion, the Beltrami flow [Kimmel00] is among the best models to process images of documents and especially ancient manuscripts (11). It preserves all information necessary for the OCR and reduces efficiently the noise of the ink and the paper. It also reinforces a little the continuity of lines.

$$(11) \quad \frac{\partial I}{\partial t} = \frac{1}{\sqrt{(1+\lambda_+)(1+\lambda_-)}} \operatorname{div}(D\nabla I) \quad \begin{cases} \mu_+ = \sqrt{\frac{1+\lambda_+}{1+\lambda_-}} \\ \mu_- = \sqrt{\frac{1+\lambda_-}{1+\lambda_+}} \end{cases}$$

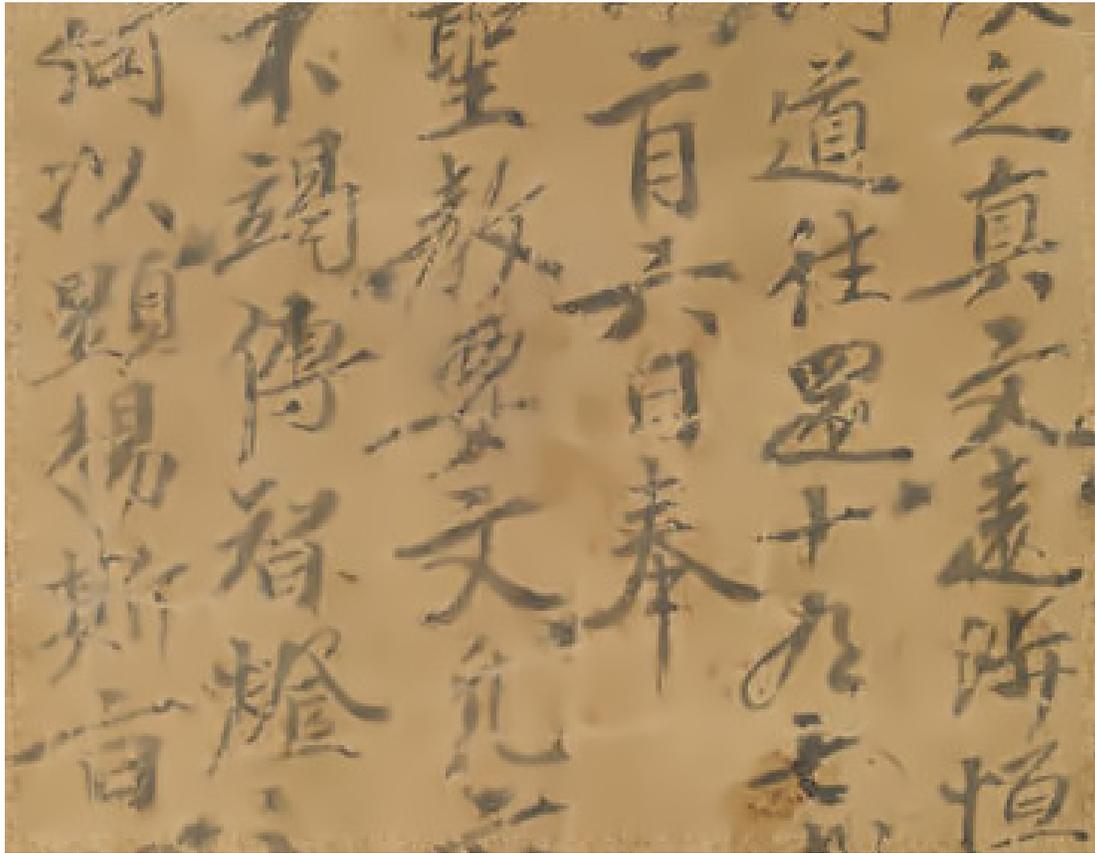


Figure 10: Anisotropic diffusion of Beltrami

Among all existing models, the Beltrami flow performs perfectly well. Among all anisotropic diffusion models presented previously, it is the more efficient model. We advise to use the Beltrami flow for the reduction of the noise when there are not too many discontinuities. In the case of large discontinuities, we advise to use the anisotropic diffusion which preserve singularities and reinforce the coherency of lines.

## 3.2 Colour segmentation and enhancement

Our objective consists of testing the feasibility to segment correctly Chinese characters from degraded manuscripts. The binarization of the luminance by any local or global thresholding completely fails to retrieve erased characters by ink fading. We need to analyze the image directly in colour to keep all necessary information for a correct segmentation. Among all experimentation, we detail only successful combination of methods which provide the best results we obtain after several days of computation to tune the optimal combination of methods and the optimal parameters.

### 3.2.1 The MeanShift

The Original MeanShift (MS) is a generic unsupervised clustering procedure based on maxima of densities function of colour clusters in the colour space [Fukunaga75,90]. The MS shifts iteratively each colour of the image to the nearest mean along the gradient directions of the density function in the colour space. It is a weighted averaging technique like the bilateral or the Non Local Mean. The weights of the averaging are the distance between colours in the colour space (12). The MS segmentation process requires no explicit definition of the clusters and is based on the density estimation into a Parzen-window which makes it well-suited for dealing with noisy and uncertain data sets. The original MS can cluster correctly non Gaussian clusters like colours in the colour space.

$$(12) \quad \bar{x} = \frac{\sum_i \bar{x}_i \times \|\bar{x}_i - \bar{x}\|}{\sum_i \|\bar{x}_i - \bar{x}\|} \quad \forall x_i \in N(x) = \{x_i / d_c(x_i, x) \leq \sigma_R\}$$

The MS requires only one parameter which is  $\sigma_R$  the size of the Parzen Window for a correct estimation of the density functions of colours in the colour space. The complexity of the MS is explained by the intensive search of colour samples in the Parzen window to compute the vector oriented towards the mean. Consequently, the complexity of the MS is  $O(N^2)$  with  $N$  equal to the number of pixels of the image. Without optimization the MS will take several days to segment only one image. Several attempts have tried to decrease the algorithm complexity mainly by adding spatial information or by reducing the number of colours to shift or even by selecting a reduced number of colours to estimate the means of the density function. But all these optimization either degrade the results or do not reduce enough the processing time. We decrease the complexity of the original MS from  $O(N^2)$  to  $O(N)$  by using integral images [LeBourgeois13]. A high resolution image of 5000x4000 can be processed in only few seconds compare to several days. The resulting figure 11, show that the MS fails to retrieve erased parts of characters when the ink is fading. Pre-processing techniques must be applied to enhance the fading colours.

### 3.2.2 Multi-scale Retinex colour Enhancement (MSR)

[Rahman96] has developed for the NASA, an efficient colour enhancement algorithm called Multi-Scale Retinex (MSR). We use such an algorithm to improve the document legibility or to retrieve colour information (Figure 12).

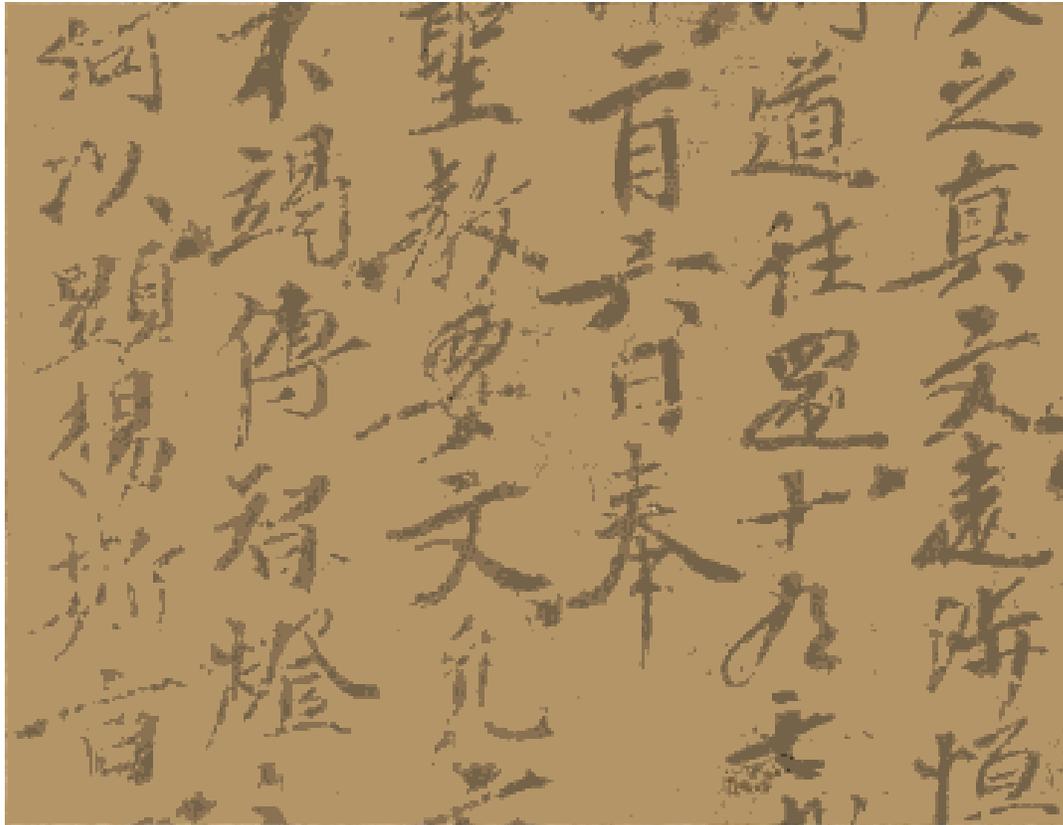


Figure 11: Segmentation by fast integral MeanShift with  $\sigma_R=7$



Figure 12: Color enhancement by Multiscale Retinex (MSR)

### 3.2.3 The MeanShift on MSR image

If we compute the MeanShift segmentation on MSR images, we obtain a better segmentation of characters. But some parts of several characters are still missing. The MSR also enhances the colorimetric noise which has been segmented by the MS. It produces false detection of patterns into the background. Moreover, noise remains along the contours of the characters and into the background.

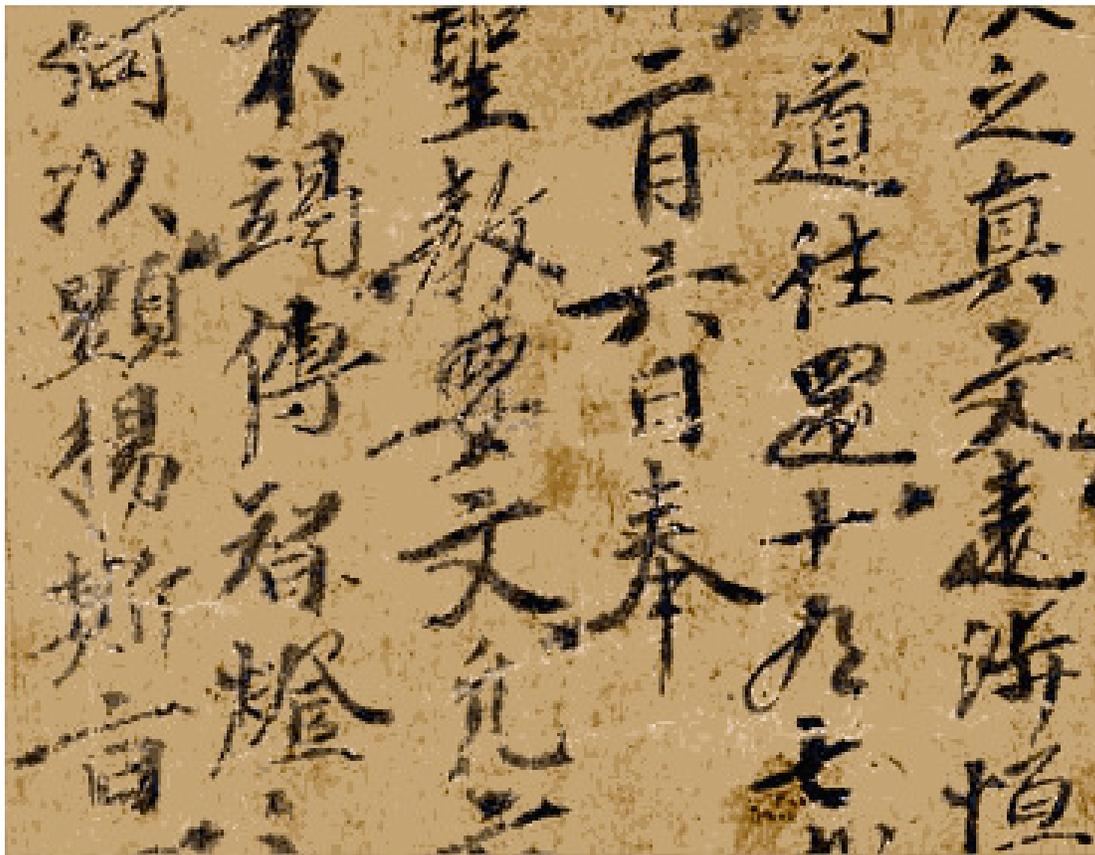


Figure 13 MeanShift on MSR Image

### 3.2.4 The MeanShift combined to anisotropic diffusion on MSR image

We propose to combine anisotropic diffusion with the MeanShift in order to simultaneously segment the image and reduce the colorimetric noise. Such work has already been successfully experimented but not yet published. We use the anisotropic diffusion which preserves edges and singularities from [Dira11] combined to the fast integral MeanShift in parallel. The result in figure14 shows real improvement of the quality of the final segmentation and the absolute limit of the colour segmentation by classical methods. When the colour of the background changes, the classical approach may fail to correctly segment the characters from stains. Figure 15 presents the success of the colour segmentation on characters written in stains.

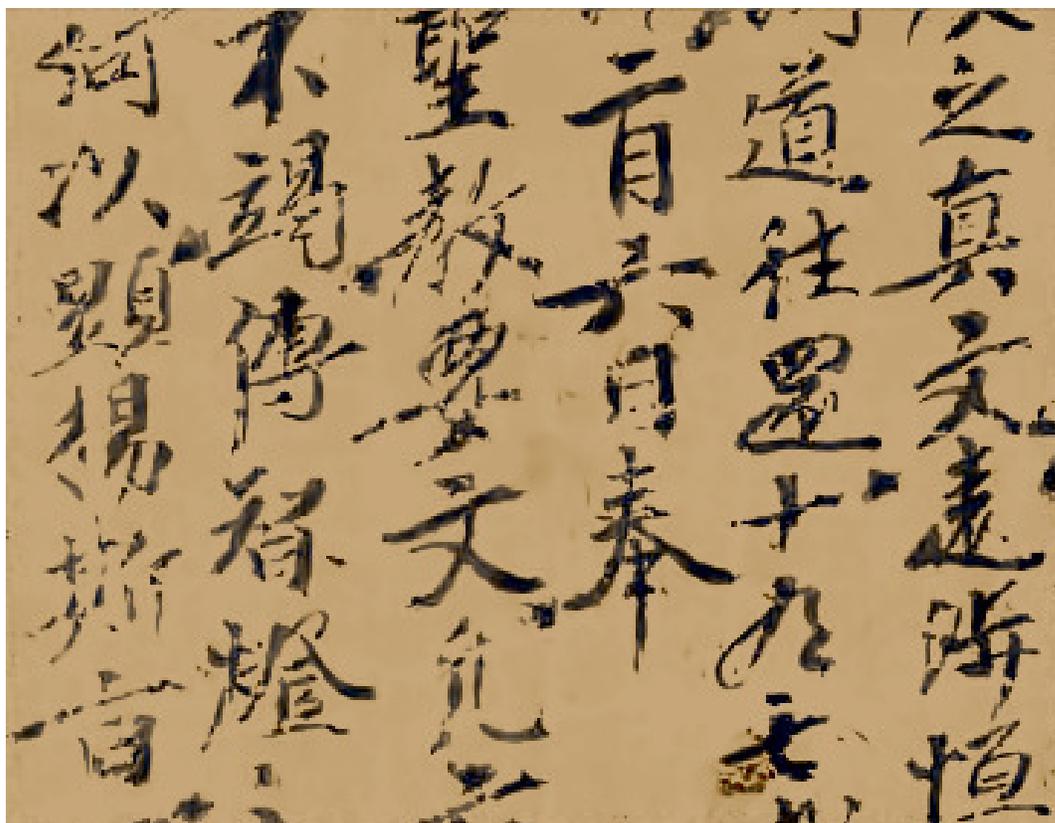


Figure 14 : Combination of MS and anisotropic Diffusion on MSR image

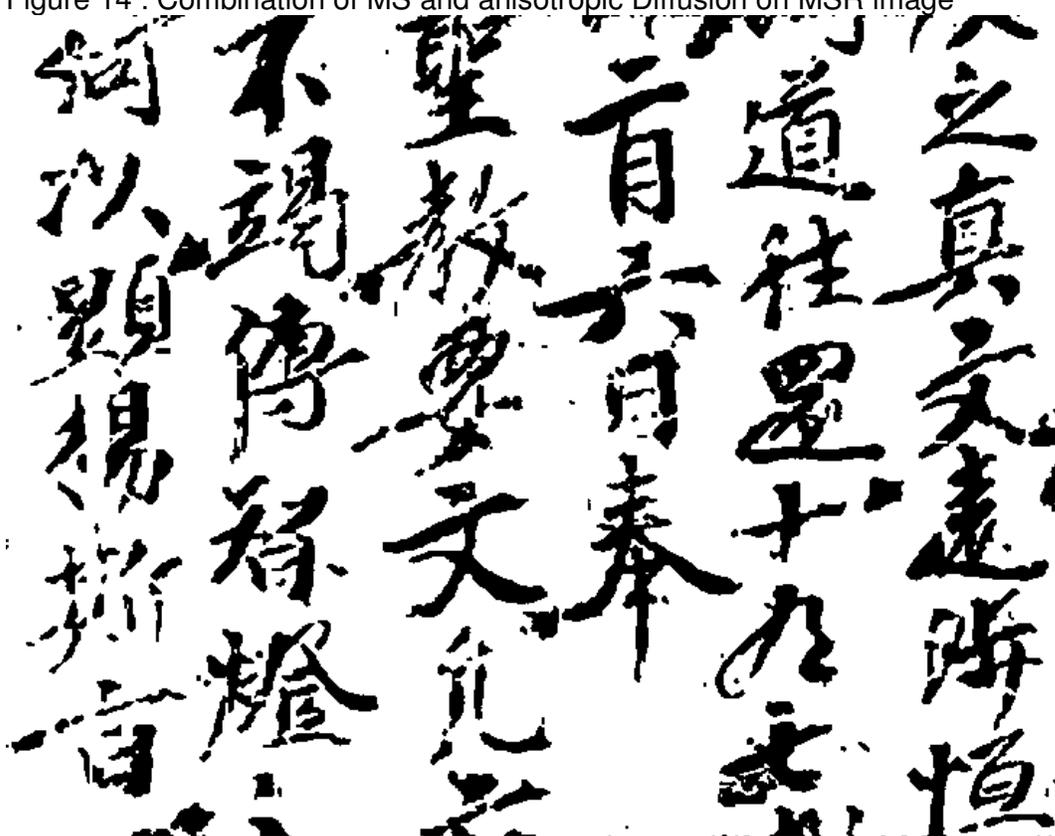
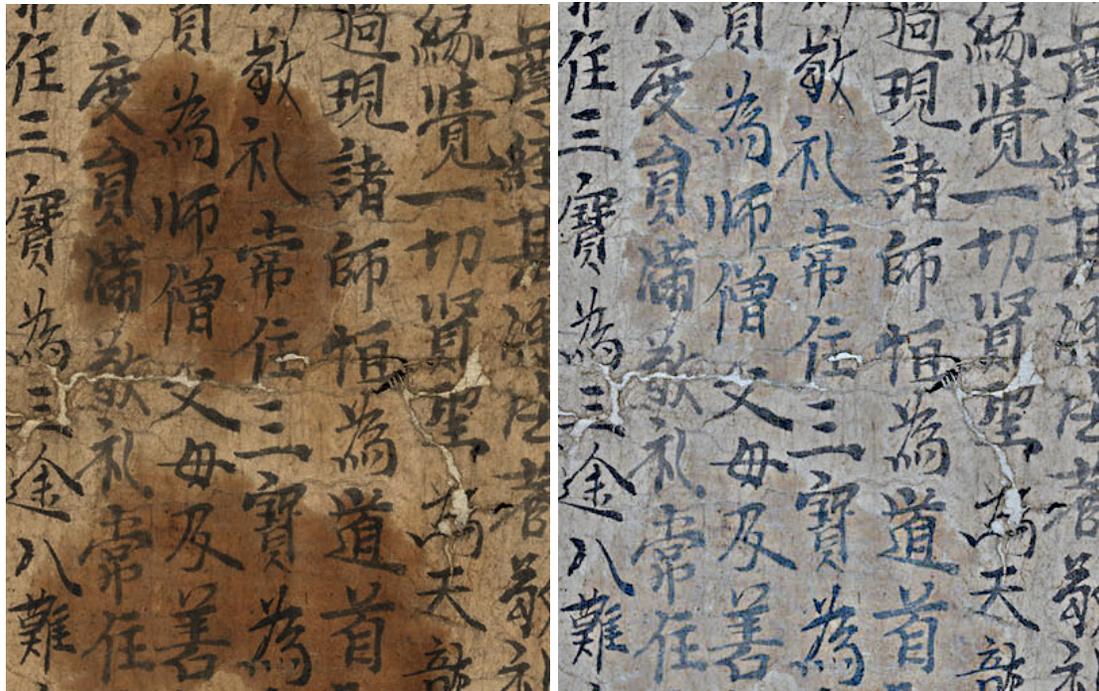
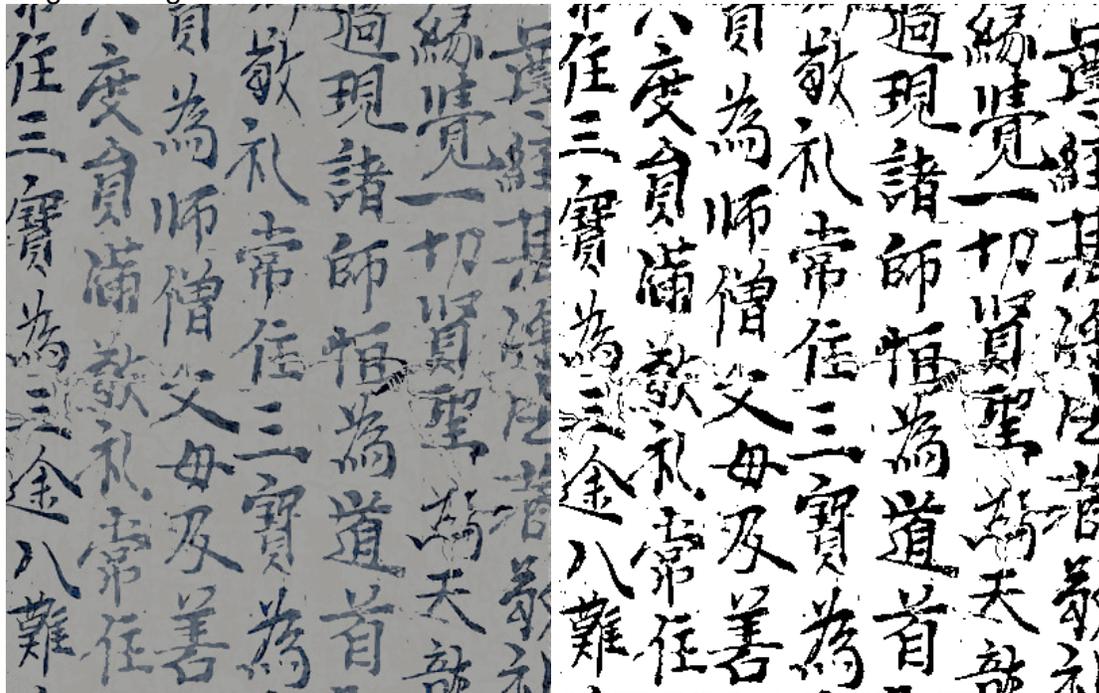


Figure 15 : Binarization of the processed image showing the character segmentation



Original image with stains

Multi-Scale Retinex Correction



Combination Anisotropic diffusion+MS

Binary images with segmented characters

Figure 16 : Result of the combination of MS and anisotropic Diffusion on MSR image with characters written inside stains of humidity.

### 3.2.5 Segmentation by Colour Morphology

In the LIRIS laboratory, we have recently developed a colour segmentation software called ACoIDPS (Robust and Unsupervised Automatic Colour Document Processing System) to segment degraded business documents. This work will be published soon. This very recent work uses mainly colour morphology and does not require any training, manual assistance, prior knowledge or model. ACoIDPS processes the images differently. It does not try to segment characters according to the colour information. It segments, by colour morphology, only thin objects whatever their colours. After the segmentation of thin colour objects, ACoIDPS finds the right colour of all segmented object by using colour information from the MeanShift operation.

We have tested this morphological process on several images of Guwenshibie project. We obtain also good performance compare to the combination of the MeanShift and Diffusion on MSR image. It processes text written into stains correctly and finds the correct colour class of characters (Figure17). Noise is also removed by morphological operations and not by diffusion. Compare to previous methods, the contours of characters are not smoothed, but the characters are well segmented.

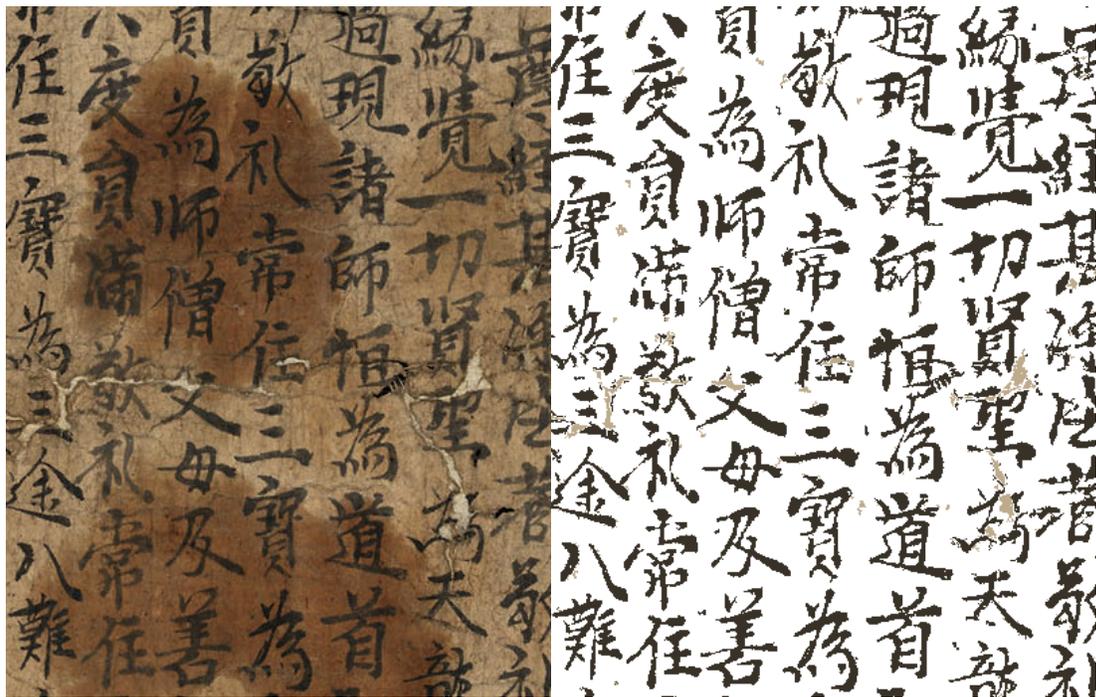


Figure 17: segmentation by ACoIDPS

### 3.3 Limits of the segmentation

In spite of all advanced algorithms, character segmentation can fail for strongly degraded manuscripts when characters ink disappear (Figure18).

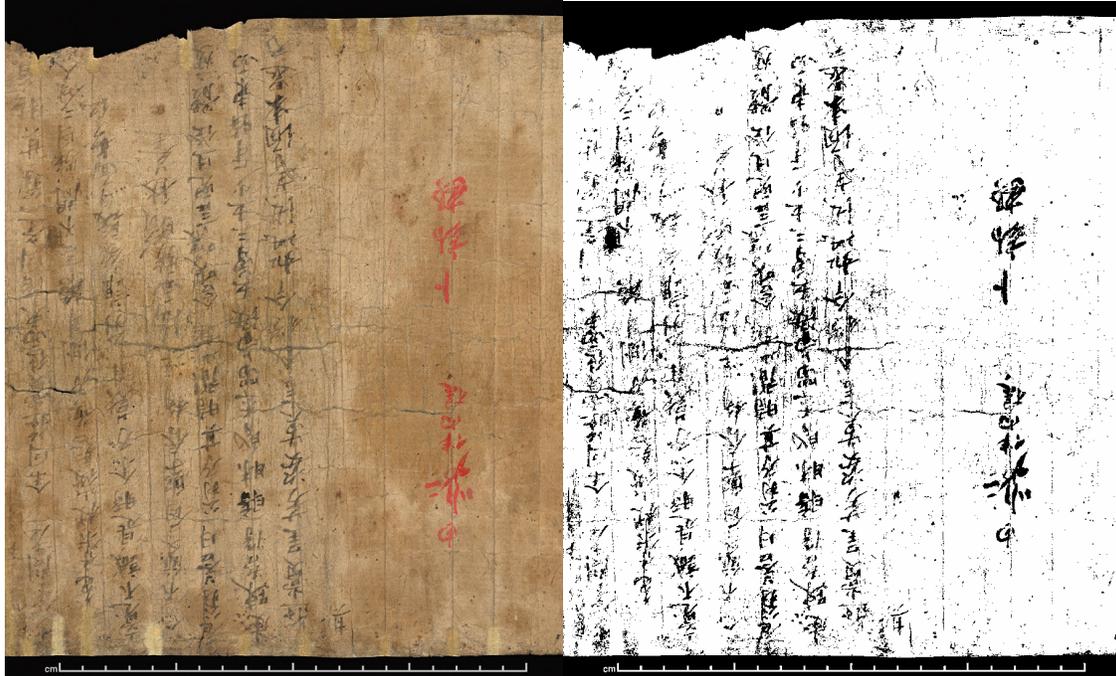


Figure 18: segmentation by Combination of MS and anisotropic Diffusion on MSR image

There is no perfect segmentation system which can well segment all type of documents whatever the degradation. It justifies the research in the other work packages on robust features and layout extraction which do not require character segmentation, image binarization and connected components analysis.

### 3.4 Automatic tuning of parameters

Most of the parameters are fixed by default to values found in the scientific literature. The others are found by experimentation. Some critical parameters are tuned automatically to optimally process images correctly. The user can manually modify some critical parameters like the  $\sigma_R$  of the MeanShift.

## 4. Software

The software Docrestorer has been developed in C++, it can process, in a workflow, thousands of images sequentially with the same parameters. The selected operations to apply are designed through the user interface. It will be freely distributed to all the partners of the Guwenshibie project for research purpose.

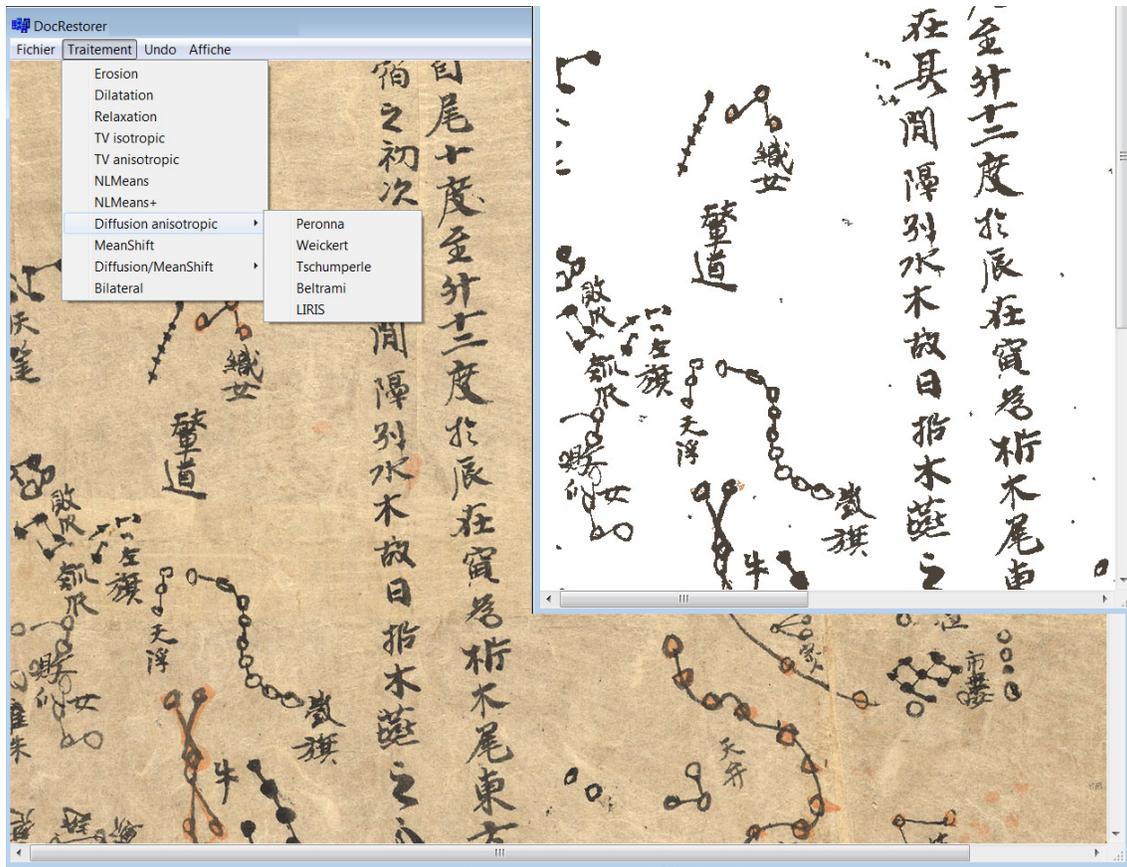


Figure 19: screenshot of the software Docrestorer

## 5. Conclusion

We have experimented only pixel-based pre-processing approaches from the literature and the main colour segmentation methods existing in the LIRIS laboratory. Most of the weighted averaging approaches do not reduce noise along the contours of the characters and cannot repair broken characters. The pre-processing operations to reduce noise, work perfectly well with variational methods like the Beltrami flow. But it can totally fail by using the Total Variation model.

The segmentation of characters by colour analysis or by morphology is feasible on degraded manuscripts. For severe degradations, when the noise magnitude is greater than the magnitude of the contours of the characters, the segmentation of characters is difficult or impossible to achieve without prior knowledge. It is also the case when the ink disappears.

The difficulties to segment correctly characters on degraded manuscripts justify the future research about new robust features and new layout extraction which do not require any segmentation.

## 6. References

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